

An accessible review on black holes: their history, formation, and accretion

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Abstract

Black holes are among the most famous scientifically studied objects. Besides the numerous papers written about them every year, they are the subjects for much of the sci-fi genre, such as *Interstellar*. Furthermore, with the relatively recent imaging of two supermassive black holes, much effort has been put into explaining black holes to the public. Yet more advanced learners—such as undergraduate researchers—are caught between the limited descriptions of public-based information and the maze of complex, deeply-mathematical papers. The goal of this paper is to bridge these two areas with an accessible yet rigorous review of black holes and their accretion disks.

Section 1: Introduction

Black holes (BHs) and the gas surrounding them embody many universal extremes. While BHs themselves are the darkest objects in the universe, the disks of gas around them can outshine entire galaxies [1]. Close to the singularity at the center, neither general relativity nor quantum mechanics alone are adequate to describe them [2]. BHs are also incredibly elusive. They were theorized to exist more than two centuries ago, but we did not have strong supporting evidence of their existence until 1964 [3]. It took a further 60 years to take pictures of two of them: Sagittarius A* (Sgr A*) and M87*. These are both supermassive black holes (SMBHs), which are present at the center of every galaxy [4]. Sgr A*

and M87* hail from the Milky Way and Messier 87, respectively. Since the images went public, there has been a growing interest in the field, highlighting a need for public BH education. Although thorough, the numerous scientific papers explaining BHs/SMBHs and their accretion are not written for a broad audience. Consequently, physicists and astronomers have endeavored to communicate their findings so that the average person can understand. Unfortunately, all this focus between the two groups has left a scientific communication chasm in the middle. We know BH review papers are perfect for high-level academics, and an article in *National Geographic* suits the public. Yet this leaves advanced learners—such as undergraduate researchers—in a predicament. The public-directed sources do not go in-depth enough for their work, while the mathematically focused papers are easy to get lost in.

Thus, this paper aims to bridge basic black hole concepts to the high-level theories of general relativity (GR) and accretion dynamics. By making the transition from public to academic smoother, we stand to make the field of compact astrophysical objects more accessible. After a brief history and relevant details of BHs/SMBHs in Section 2, Section 3 provides an in-depth look into the accretion process and how we describe BH accretion. Section 4 covers the different types of BH accretion regimes. Section 5 deals with how we see BHs and the role

accretion disks play in that process, and we conclude in Section 6 with a summary and overview of current research directions.

Section 2: A brief overview of a black hole and its lifespan

Brief History Recap

The first person to theorize the existence of a black hole was John Michell in 1784 [5]. He solved Newton's equation of gravity for the escape velocity of a particular massive object:

$$KE = W$$

$$\frac{1}{2}mv^2 = \frac{GMm}{R},$$

where we have set Kinetic Energy and Work equal to each other. Substituting their definitions in, and solving for the escape velocity v in terms of the mass of the object achieving that velocity m , the more massive body's mass M , the gravitational constant G , and the radius of the more massive body R , we have

$$v = \sqrt{\frac{2GM}{R}}. \quad (1)$$

Now, assuming the escape velocity is the speed of light c

$$c = \sqrt{\frac{2GM}{R}}$$

$$\frac{c^2}{2G} = \frac{M}{R}, \quad (2)$$

or that the ratio of mass to radius is equal to $\frac{c^2}{2G}$. If $\frac{M}{R}$ is any larger, then the body in question would have an escape velocity greater than the speed of light. Michell called them 'dark stars'. Unfortunately, this idea was soon abandoned. With Thomas Young's discovery of light as a wave, many were confused about how a wave could be influenced by gravity, if at all [5]. Thus, the topic lay untouched for 131

years, until a certain Swiss patent clerk came along. Albert Einstein published his governing equations for general relativity (GR) in 1915; unbeknownst to him, the idea of a 'dark star' lay within them. It was uncovered when Karl Schwarzschild, a German Physicist fighting on the Russian frontlines of WWI, solved Einstein's field equations for a point mass. He found that past a specific distance, now called a Schwarzschild radius, not even light could escape the point mass' pull. This radius is quite easily obtained from our earlier equation:

$$R = \frac{2GM}{c^2}. \quad (3)$$

If one were to pass this "one-way" membrane, they would be forever severed from the rest of the universe. Today, this radius is referred to as the *event horizon*. Note that we assume a spherically symmetric, nonrotating solution for these equations to work. Schwarzschild's full solution to the field equations forms the *Schwarzschild metric*. Further, more accurate/applicable metrics have been developed, such as the Kerr metric for a spinning BH. Nevertheless, for our purposes, we shall only need applications described by the above equations.

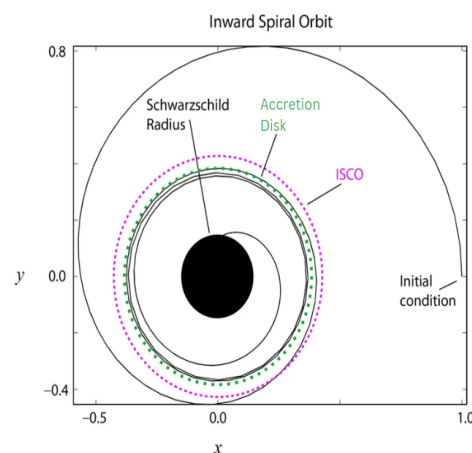


Figure 1: A diagram showing the trajectory of a particular photon into the black hole. Here included are the radii for the ISCO and Event horizon/Schwarzschild radius, as well as the innermost radii of a typical accretion disk (from [6]).

The term *black hole* did not come about until 1967, coined by physicist John Wheeler [5]. It is an arguably more descriptive name than ‘dark star’: any light going into a BH can never bounce off and reach us, rendering it a ‘black hole’ in space. Following that same thread, one might ask how we may observationally demonstrate a BH’s existence. The answer lies in the effects of a BH on its surroundings. According to GR, light follows straight paths in spacetime. If enough mass is present in a given area, spacetime will bend around it, causing light rays to follow suit and appear to ‘curve’ around the object. An excellent example of this comes from Fig. 2, shown below.

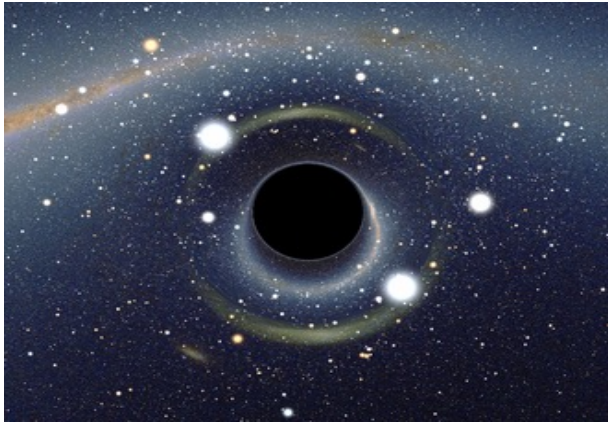


Figure 2: A computed image of a black hole in front of the Large Magellanic Cloud. Notice that a singular star now looks like two thanks to gravitational lensing (top left & bottom right). Adapted from [7].

Another way to detect a BH is to look at its interactions with neighbors. Due to a BH’s immense gravitational field, it can rip stellar material off a nearby star. As we will discuss in the following sections, this process (called accretion) generates electromagnetic radiation of a specific range. According to NASA, this is how astronomers first proved the existence of the BH Cygnus X-1. “Astronomers saw the first signs of the black hole in 1964 when [they] detected celestial sources of

¹ A solar mass is the mass of the sun, denoted as M_{\odot} .

² Degeneracy pressure is the resistance of too many (alike) fermions being too close together. In cases where electrons/neutrons are squeezed together, they get ‘claustrophobic’ and resist compression.

X-rays.... In 1971, astronomers determined that the X-rays came from a bright blue star orbiting a strange dark object. It was suggested that the detected X-rays resulted from stellar material being stripped away from the bright star and ‘gobbled’ up by the dark object—an all-consuming black hole” [8]. Since then, we have photographed two of them millions of lightyears away. From contemplating dark stars to seeing the unseeable, there has been tremendous progress in the relatively short 200-year span of the field.

Scientific Background

Our sun, and every other star in the universe, is embroiled in a fierce war between two forces: the pressure of fusion and gravity. Each star starts with a certain amount of ‘nuclear fuel’ when it is born, primarily made up of the lightest elements such as H and He. Nuclear fusion happens when two lighter elements, e.g., H with itself, come close enough to fuse and produce a heavier element like He. This process happens once every 1.1×10^{-38} seconds in a star, releasing 4.26 million metric tons of energy every second [9]. This consistent outflow of energy pushes against the gravitational force that pulls the star inwards.

What happens when the fuel runs out? Without fusion holding it back, gravity will soon collapse the star to a smaller radius, and it would keep going if not for other forces in play. If, after the initial crunch, the remaining mass contains below 1.44 Solar masses¹, electron degeneracy pressure² will take up arms against gravity and stop further compression [10]. After that but before 3 M_{\odot} , neutron degeneracy pressure becomes the last line of defense. Further than that, however, gravity becomes too strong and overpowers all of them. It will continue squeezing the star until its mass is

compressed into a single point (called a singularity), thus finally becoming a black hole.

Although the previous paragraph describes the formation of stellar-mass black holes, it does not hold for SMBHs. No star could ever grow large enough to collapse into a SMBH [3], so astrophysicists still are still unsure how they form. Some theories have attempted to explain their existence, such as mergers of intermediate-sized BHs, collapses of multiple stars simultaneously, and even large clusters of dark matter [3]. Nevertheless, the exact theories and research behind SMBH creation are beyond the scope of this review. For our analysis, we will be content with the evidence that they exist.

Black Holes Don't Have Hair

The reason why it is so difficult to deduce the origins of BH formation is the exact reason why studying them can be so straightforward—black holes do not have hair. This comes from the famous “black hole hair” theorem; ‘hair,’ as it relates to this argument, is any distinguishing information about the object. When a star implodes to become a BH, it ‘cleans’ itself of all its previous history. Anything about the former star’s surface temperature, luminosity, size, and more is lost after the BH becomes stable [11]. Twenty different stars could implode and become the same black hole, which is problematic for those interested in BH formation but incredibly helpful to those studying its present activities. To describe any BH, no matter where or when in the universe, all one needs is its charge, angular momentum, and mass. Even better, most astrophysical³ BHs either have a negligible charge or none at all. We will use these ideas fully for further sections.

³ An ‘astrophysical’ black hole is simply one that is possible, or most likely to be found in space. The following sentence states most observed black holes do not harbor any electrical charge.

The Death of a Black Hole

If material, once sucked into a BH, could not leave, then one would think BHs never lose any mass. With their prodigious appetites and ability to combine, it seems like these astrophysical behemoths are here to stay forever. However, there is one way they might ‘die,’ Hawking radiation [12]. Discovered by Stephen Hawking in the 1970s, Hawking radiation is a process that states a BH will evaporate away its mass M in a time proportional to $(M/M_{\odot})^3$.

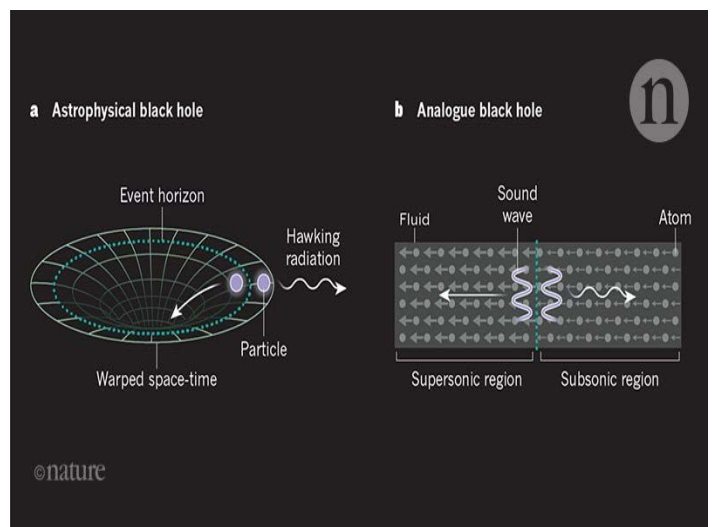


Figure 3: A diagram of the process of Hawking radiation. The purple circles are virtual particles, and the inwards funnel represents the potential energy well of the black hole as a physical well. The blue dotted line is the event horizon, where there is no coming back from. Adapted from [13].

The concept of virtual particles arises in quantum mechanics when describing ‘empty space.’ Space is not actually empty but full of virtual particles—matter-antimatter pairs that spontaneously come into existence and soon thereafter annihilate each other to conserve total energy. However, close to a black hole, the outcome differs significantly. As shown in Fig. 3, when the virtual particles pop into existence, one gets sucked

in by the black hole while the other escapes. This induces negative energy into the black hole, and from $E = mc^2$ we know that it also correlates to a negative mass [14]. Thus, if no other material accretes, the BH will decrease in mass until it completely evaporates. However, this evaporation is mainly insignificant; most BHs will take millions of megayears to radiate away all their mass. Moreover, this process assumes that no matter accretes in the meantime, an entirely inaccurate approximation for almost every astrophysical BH.

Section 3: What is accretion?

With a strong background in BHs, we may now move into a common process they partake in, *accretion*. Accretion is the “inflow of matter toward a central gravitating object or toward the center of mass of an extended system” [8], not just toward a black hole. In general, accretion is responsible for the formation of galaxies, solar systems, and much more. Specifically, black hole accretion is the inflow of matter—usually stars/interstellar material made up of gas—that forms a disk around a black hole [15]. We denote the accretion rate as $\dot{M} \equiv \frac{dM}{dt}$, the rate of change of the BH’s mass M . When accreting, very little of the gas has velocities directing them straight into the BH. Instead, most have trajectories that put them in a circular (or elliptical) orbit, along with other accreted material around the BH.

Since angular momentum \vec{L} is equal to $\vec{r} \times \vec{p}$, where the momentum of the infalling object $\vec{p} = m\vec{v}$ and is perpendicular to \vec{r} , then $L = Rmv$ for a radius of $r = R$. Using the right-hand rule on the figure above, we can determine that L is pointing ‘out of the page’. Furthermore, we can use Eq. 1 to simplify this expression:

$$L = Rmv = Rm \left(\sqrt{\frac{2GM}{R}} \right) = m \sqrt{2GM} * R^{\frac{1}{2}}$$

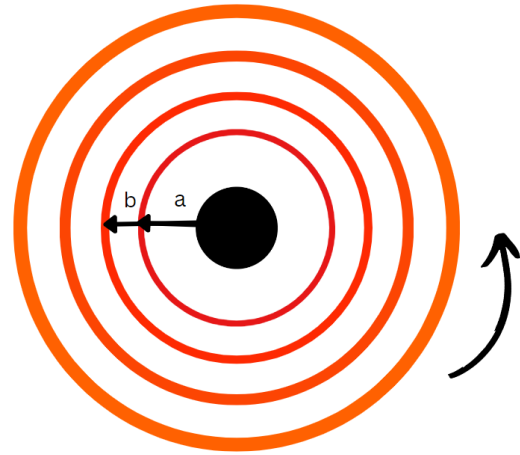


Figure 4: A diagram of a simplistic accretion disk, where each ring of an equal mass of gas is traveling with the same speed at the same radius. It is convention that top-down views of an accretion disk be shown such that the matter is orbiting counterclockwise. Here, a and b are radii ($b > a$).

Thus, as the colors go from orange to red in Fig. 4, L goes from a large to small value. This result should make sense: since angular momentum is proportional to \sqrt{R} , then as the radius increases, the angular momentum increases. This is why ring b has higher angular momentum than ring a . These results are dependent on our earlier assumptions that the motion is *Keplerian*⁴ and the mass of each ring is equal. If that does not hold, the previous equation must be modified for certain accretion disk models where other forces become consequential.⁵ Nevertheless, we see that the accreted material must decrease its angular momenta to move closer to the BH.

But how does this happen? Indeed, those who have taken introductory physics would see no reason for it to change orbits. After all, Earth is undergoing circular motion around the sun, governed by (mainly) the same equation(s) as above. So why should we expect gas at any radius to move closer to the BH? To see why, let us consider the

⁴ We say motion is *Keplerian* if we only consider gravity for two bodies, and the mass of one is much greater than the other such that the smaller object exhibits circular motion around the larger one.

⁵ See section 5 for more details.

following example. Two cars are traveling down the highway. One is moving faster than the other, but for a noticeable window of time, they come into contact with each other. As a result, friction will be generated between the cars. Their sides will heat up, and sparks will fly between them. This is, in fact, analogous to an accretion disk. Looking back to Fig 4., we can break the accretion disk into discrete rings. Like in our example, these consecutive rings traveling at different speeds will experience ‘friction’ between them, causing them to start heating up and transferring angular momentum between them.⁶ This momentum transfer is a critical topic in accretion disk theory and is why anything undergoing circular motion would ever fall into a BH.

As rings of matter lose angular momentum, they move inwards. It follows from Eq 1. that the ring must increase its velocity, but from our earlier example, this also increases the magnitude of friction between consecutive rings. So as the rings move closer to the BH, they lose angular momentum and decrease their orbit. A small fraction of gas will move to larger radii to conserve net angular momentum and avoid becoming the BH’s next meal. Eventually, the infalling ring of gas reaches a radius of which it can no longer undergo circular motion. Unlike Newtonian mechanics, GR states that there is an innermost stable circular orbit (ISCO) around a BH. At the ISCO, radial orbits are no longer possible, for the object’s velocity would be greater than the speed of light; thus, at the ISCO matter must plunge into the BH.

Therefore, we conclude that BH accretion involves material (usually gas) moving in rings around it, where an analog to friction causes each ring to decrease its angular momentum and eventually plunge into the BH. Early on, it was thought that the friction due to molecular interactions between rings created all the light we

see. However, when theorists did the calculations, they found it insufficient to produce the light we observe. It turns out that magnetic fields are usually necessary to create enough “friction” to accrete efficiently [16]. The details of the accretion process—such as how much gas is in the disk, how exactly consecutive ring friction acts, how fast the gas falls into the BH, etc.—keep this from being a general process. Instead, one can choose different approaches through accretion disk models.

Section 4: Accretion disk models

With the motivation and theory in place, we may move on to accretion disk (AD) models. Similar to stellar theory, there is no ‘one theory fits all.’ Instead, the disparate astrophysical environments of black holes are analyzed separately in *regimes*. Density, pressure, rotation, accretion rate, and more are all used to create the regimes of BH AD theory. There are several which are prominently used, as seen in Table 1. This paper will only cover the thin, slim, and ADAF models.

	Fast rotation (Disk)	Slow rotation (Bondi)
Large pressure	slim, thick	ADAFs
Small pressure	thin	free-fall
	Accretion rate high	Accretion rate low
Large opacity	slim, thick	thin
Small opacity	–	ADAF

Table 1: This table from [18] describes the different regimes of accretion disk theory, and what theories occupy those spaces. Notice that some solutions, like ADAF, are not limited to one group. Nor is a grouping limited to one model, e.g. Large pressure & Fast rotation.

Shakura-Sunyaev (SS)/thin disk

The first widely used disk model, hereafter called the SS disk, was created in 1973 [17]. Friction due to molecular interactions between gas particles was insufficient to explain the observed luminosities. Therefore, we needed a model to explain or parameterize the friction that leads to angular momentum loss—the arbiter of accretion.

⁶ For more detailed descriptions, see [14].

Shakura and Sunyaev developed a parameter, α , to parameterize viscosity (the AD analog to friction). We present a few equations that govern SS disk dynamics to motivate further learning:

The law of mass conservation is

$$2\pi r \Sigma v_r = \dot{M},$$

where Σ is the mass surface density in the disk, and \dot{M} is the accretion rate, or change in mass of the annulus (disk) at radius r .

The law of angular momentum conservation is

$$v_\phi = \frac{GM}{r},$$

where v_ϕ is the azimuthal velocity.

In deriving these formulas, it was necessary to make assumptions about the environment. Namely: the disk is in steady-state, local thermal equilibrium, the ratio of height to radius is much less than one ($h(r)/r \ll 1$), it possesses axial symmetry, and all heat is immediately dissipated as light.

The SS disk has succeeded in describing certain types of BH accretion flows where they hold. Such is the case in Quasars, an extremely luminous active galactic nucleus (AGN).⁸ While limited in scope due to the model's assumptions, it was nevertheless a critical first step in the development of accretion disk theory.

Slim disk

One situation where the SS disk can be ill-equipped to handle is a BH where the accretion rate \dot{M} is high. To quantify this, we compare it to the *Eddington rate*. The Eddington rate, \dot{M}_{edd} , is a rough approximation of the maximum accretion rate a compact object can have. At \dot{M}_{edd} , the outward pressure of radiation would exceed the BH's gravitational pull on the gas, halting accretion

⁷ Hence why it is also called the thin disk model.

⁸ An actively accreting SMBH at the center of a galaxy.

⁹ Advection is the transfer of heat/matter by the flow of a fluid.

entirely [15]. When we have close to Eddington rates

($\frac{\dot{M}}{\dot{M}_{\text{edd}}}$ close to 1), the disk becomes thick enough

to move energy around from one radius to another through a process called advection⁹ [18]. We can no longer assume Keplerian motion, either. Thus, a new model was created called a slim disk. Started in the 1980s by Abramowicz [16], this model does not assume the disk is geometrically thin. Instead, it can handle disks where H/R is 0.3 [18]. These disks can get so dense that light from the midplane has trouble escaping the disk. We would call this 'optically thick,' where the gas density prohibits most emissions from escaping. Back to advection, most of the heat generated by viscosity/friction is "advected inward and released closer to the black hole or not released at all" [18]. As a result, such disks' luminosities are lower than expected for their accretion rate.

Shown below are a few equations that govern Slim disk dynamics:

The law of mass conservation is

$$-2\pi r \Sigma v_r = \dot{M},$$

where the constants are the same as in the previous subsection.

The law of angular momentum conservation is

$$v_\Sigma = \frac{\dot{M}}{3\pi} f * g^{-1},$$

where $\Sigma = 2Hp$ is the surface density,

$f_* = 1 - 9\Omega/(\Omega(R/R_G)^2)$ and $g = -\frac{2}{3} * \frac{d \ln(\Omega)}{d \ln(R)}$ [15].

Furthermore, these solutions are valid all the way to the event horizon, as opposed to the ISCO for SS disks. It is also applicable to some BH X-Ray binaries [18].

Advection Dominated Accretion Flow (ADAF)

At very low accretion rates ($\frac{\dot{M}}{\dot{M}_{\text{edd}}}$ much less than 1), the gas can no longer cool efficiently. Instead, the primary mechanism that gas cools becomes advection. For a BH, if the gas falls in quicker than it can emit, it is advection-dominated. First explored in the 1970s [19], an advection dominated accretion flow (ADAF) for a BH is when we consider an AD that cools exclusively by advection. The solution is optically thin, allowing radiation to escape easily. The motion is usually non-Keplerian, though it is not strictly prohibited [18]. Furthermore, the radiative efficiency decreases as \dot{M} increases. They are associated with jets as well [18].

The geometric shape of the accretion disk is similar to a sphere/corona, and the disk is very hot. Because they are advection dominated, the matter falls into the BH before it can radiate—rendering them less luminous than the SS disks.

It is believed that both Sgr A* and some BH X-Ray binaries can be described by an ADAF solution [19].

With these three main AD models, we can characterize ADs around most BHs. Moreover, we can also make predictions on BH observations, such as apparent luminosity and spectra. But what exactly is it that we observe from a ‘black hole’?

Section 5: How can we see a ‘black’ hole? Radiative processes and how they produce black hole spectra

Black hole spectra are the electromagnetic spectrums one observes from a black hole. They can differ significantly based on accretion disk qualities, which is why different BHs may have different spectra. The most common type of emission is thermal emission or thermal light. However, when the disk is not in thermal equilibrium (not optically thick), we see light created by *emission mechanisms*.

These mechanisms rely on Larmor’s law, which states that accelerating charges produce changing electromagnetic fields. In other words, photons. From this rule, two primary processes create photons: *Bremsstrahlung* and *Synchrotron radiation*.

Bremsstrahlung

Assume an electron is moving along a particular path through space. We know that electrons are affected by electric fields; specifically, they are attracted to positive charges. If one were to present itself suddenly to the electron, it would change its velocity to move closer to the electric field’s source. Of course, that change in velocity can be described by an acceleration. Finally, by Larmor’s law (accelerating charges produce changing electromagnetic fields), we know this situation has created photons. This situation is described as *Bremsstrahlung* (German for ‘braking radiation’). It occurs when an electron gets close to a proton or nucleus and changes its trajectory entirely, causing it to emit a photon. It is important to note that an accretion disk is mainly made up of gas, which itself is primarily composed of electrons, protons, and ions. Thus, the amount of *Bremsstrahlung* one observes depends on the density squared, and it becomes clear to see how this situation frequently occurs in many accretion disk models.

Synchrotron radiation

For a more consistent example, consider an electron undergoing circular motion. Although the magnitude of its velocity does not change, the direction does, meaning that it is accelerating. As aforementioned, by Larmor’s law, we know this will produce photons outwards. Unlike the previous examples of gravity providing the centripetal force, however, it is the magnetic force in this case. Denoted as *Synchrotron radiation*, it is one of the most prevalent processes producing electromagnetic radiation in the universe. It is also prevalent in ADs

due to the strong magnetic fields in most disks.

Beyond these, there are also ways that light can move across the disk, called *scattering mechanisms*.

Compton/Inverse Compton scattering

We know momentum must be conserved when a high-energy photon collides with an electron. Since this is a perfectly elastic collision, the two separate particle momenta come closer together in value. Hence, the electron gains energy off the rebound, while the photon loses that same amount of energy. This process is called Compton scattering. Inverse Compton scattering proceeds similarly, but with one small twist—the electron switches roles with the photon. In other words, the electron has a higher energy and transfers some of it to the photon.

Fig. 5 provides a closer look at how one BH characteristic (accretion rate) influences the spectrum we observe.

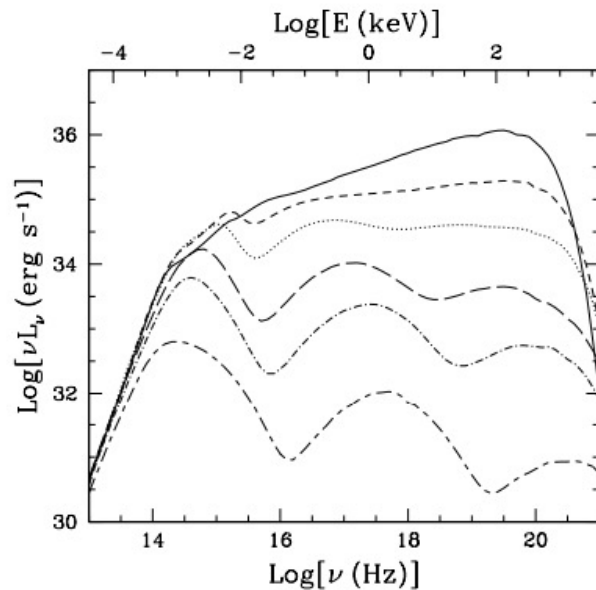


Figure 5: Graphs of a black hole's spectra, with $M_{\text{BH}} = 10M_{\odot}$. The vertical axis is plotted on a logarithmic scale of the total photon luminosity, with photon frequency on the horizontal axis. The lines go from low to high accretion rates, with the first bump corresponding to a synchrotron emission, and inverse Compton scattering for the second and third bumps. Note that for higher accretion rates, the amplitudes of these bumps increase such that for the top line, the whole spectrum becomes one semi-straight line. Plot from [20].

Altogether, these processes are called *radiative transfer mechanisms*. Knowing which mechanisms are most prevalent in the spectra can help astrophysicists determine the characteristics of the accretion flow and hence the AD model to use. For example, if X-ray emissions dominate the spectra, it is most likely a thin disk. Low luminosity? An ADAF seems like the perfect choice. And so on. Thus, by characterizing the spectrum, we can model the source's AD and use it to predict future changes in spectra.

Section 6: Conclusion

This paper has covered black hole concepts a level above publicly available material. First, giving a background on a black hole's birth, life, and (possible) death, we dove into a critical process in its life—accretion. Next, we covered what accretion is and how we describe it. To that end, we have explored the different regimes of accretion disk theory and their related models and briefly introduced the mathematical formulas behind them. Finally, we discovered their applications when discussing black hole spectra and their use in confirming/predicting observations. This paper has therefore bridged the gap between publicly available material and the deeply mathematical papers regarding black holes, educating and inspiring advanced learners to research further.

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